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AN EQUATION FOR RAPID CALCULATION OF STAGNATION POINT RADIATIVE HEAT TRANSFER (INCLUDING SHOCK LAYER RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION)

12 APRIL 1968

NOL

UNITED STATES NAVAL ORDNANCE LABORATORY, WHITE OAK, MARYLAND

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AN EQUATION FOR RAPID CALCULATION OF STAGNATION POINT RADIATIVE HEAT TRANSFER (INCLUDING SHOCK LAYER RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION)

by

John D. Anderson, Jr.

ABSTRACT: A closed-form equation is derived for stagnation point reentry radiative heat transfer accounting for the combined effects of radiative cooling and nongray self-absorption within the shock layer. The equation can be applied for both continuum and atomic line radiation. In addition, the equation is shown to agree favorably with existing numerical data for stagnation point, continuum, radiative heat transfer for a wide variety of conditions. Also, the equation is shown to apply to the end-wall radiative heat transfer behind a strong reflected shock wave in a shock tube. Finally, the equation provides a rapid means of obtaining, by hand, reasonably accurate engineering estimates for reentry radiative heat transfer including shock layer radiative cooling and nongray self-absorption.

U. S. NAVAL ORDNANCE LABORATORY
White Oak, Silver Spring, Maryland

NOLTR 68-56 12 April 1968

An Equation for Rapid Calculation of Stagnation Point Radiative Heat Transfer (Including Shock Layer Radiative Cooling and Nongray Self-Absorption)

This report presents a closed-form equation for stagnation point reentry radiative heat transfer accounting for the combined effects of radiative cooling and nongray self-absorption within the shock layer. The equation can be applied for both continuum and atomic line radiation. In addition the equation is shown to agree favorably with existing numerical data for stagnation point, continuum, radiative heat transfer for a wide variety of conditions. Also, the equation is shown to apply to the end-wall radiative heat transfer behind a strong reflected shock wave in a shock tube.

This project was performed for Foundational Research under FR-61.

E. F. SCHREITER
Captain, USN
Commander

L. Schindel

L. SCHINDEL
By direction

CONTENTS

Page

INTRODUCTION 1

ANALYSIS 3
Background 3
Vacuum Ultraviolet Contribution 8
Long Wave Length Contribution 13
Application of Formula 16

RESULTS 18

CONCLUSIONS 23

REFERENCES 26

TABLE I
Tabulated Information from Engineering Formula and Comparison with Numerical Results 21

ILLUSTRATIONS

Figure	Title
1	Effect of radiative cooling and nongray self-absorption on stagnation point radiative and convective heat transfer. V_∞ = 50,000 ft/sec; alt = 200,000 feet
2	Effect of radiative cooling and nongray self-absorption on those portions of stagnation point radiative heat transfer above and below 1100Å. V_∞ = 50,000 ft/sec; alt = 200,000 feet
3	Effect of radiative cooling and nongray self-absorption on the stagnation region shock layer enthalpy profile. V_∞ = 50,000 ft/sec; alt = 200,000 feet
4	Schematic of nongray continuum step model absorption coefficient.
5	Effect of radiative cooling and nongray self-absorption on the distribution of $B_1\epsilon_2$ through the stagnation region shock layer as a function of τ_1. V_∞ = 50,000 ft/sec; alt = 200,000 feet; R = 2.0 feet
6	Variation of $B_1\epsilon_2$ with τ_1 through the stagnation region for various nose radii. V_∞ = 50,000 ft/sec; alt = 200,000 feet
7	B_1 and B_2 as functions of temperature.
8	$f(\Gamma_{eff})$ as a function of Γ_{eff}.
9	Comparison of engineering formula with numerical results for stagnation point continuum radiative heat transfer.
10	Comparison of engineering formula with measured shock tube end-wall radiative heat transfer rates behind a reflected shock wave.

NOMENCLATURE

B_λ	Planck black body function $= 2hc^2\left[\exp(hc/\lambda kT) - 1\right]^{-1}\lambda^{-5}$
B_1	$\int_0^{1100Å} B_\lambda d\lambda$
B_2	$\int_{1100Å}^{\infty} B_\lambda d\lambda$
E_s	total radiative energy emitted per second per unit volume evaluated for equilibrium normal shock conditions
E_{1s}	radiative energy emitted per second per unit volume between 0 and 1100Å and evaluated for equilibrium normal shock conditions
E_{2s}	radiative energy emitted per second per unit volume above 1100Å evaluated for equilibrium normal shock conditions
Q_R	total stagnation point radiative heat transfer
$Q_{R_{0-1100}}$	stagnation point radiative heat transfer below 1100Å
$Q_{R_{1100-\infty}}$	stagnation point radiative heat transfer above 1100Å
R	nose radius
T_s	equilibrium temperature behind a normal shock
V_∞	free stream velocity
y	distance measured normal to the surface in the stagnation region
δ_{AD}	shock detachment distance for the shock layer without radiative cooling
ϵ_2	exponential integral of the second order; $\epsilon_2(t) = \int_0^1 \exp(-t/w)dw$
ε	ρ_∞/ρ_s

Γ	radiation loss parameter, Eq. (6)
Γ_{eff}	effective radiation loss parameter, Eq. (12)
Γ_1	effective radiation loss parameter for the v-u-v region
Γ_2	effective radiation loss parameter for the long wave length region
κ_1	v-u-v absorption coefficient, Eq. (1)
κ_2	long wave length absorption coefficient, Eq. (2)
κ_λ	spectral absorption coefficient
λ	wave length
ρ	density
τ_1	v-u-v optical length = $\int_0^y \kappa_1(y)dy$
τ_{1s}	characteristic v-u-v optical thickness of the shock layer = $\kappa_{1s}\ \delta_{AD}$

Subscripts

1	v-u-v wave length region
2	long wave length region
λ	per unit wave length
∞	free stream conditions
AD	without radiative cooling
s	equilibrium conditions immediately behind the normal bow shock wave

INTRODUCTION

Radiative heat transfer from the high temperature shock layer about a large, blunt, superorbital reentry vehicle is strongly influenced by the combined effects of radiative cooling and nongray self-absorption within the shock-heated gas. In fact, recent analyses[1-3] have shown that, for superorbital reentry velocities, radiative cooling and nongray self-absorption can reduce the stagnation point radiative heat transfer by as much as an order of magnitude in comparison to predictions based on a constant property, transparent gas. Similar effects have been noted on the high temperature gas behind a strong reflected shock wave.[4,5] Unfortunately, in order to take these effects into account, the above analyses have required detailed and tedious numerical calculations of the radiating shock layer. This situation prompts the following question: can a simple, approximate, closed-form equation be derived which would allow rapid but accurate engineering calculations of stagnation point radiative heat transfer <u>taking into account</u> the combined effects of radiative cooling and nongray self-absorption? The practical benefit of such an engineering formula, namely, to circumvent lengthy numerical calculations, is obvious. In answer to the above question, the present paper presents a rational, physical derivation of such a closed-form equation for stagnation point radiative heat transfer accounting for radiative cooling and nongray self-absorption. This equation can be applied for both continuum and atomic line radiation. In addition, the resulting formula is shown to agree favorably with existing numerical

data for stagnation point, continuum, radiative heat transfer for a wide variety of conditions. Also, the formula is shown to apply to the end-wall radiative heat transfer behind a strong reflected shock wave in a shock tube.

The present analysis is an outgrowth of previous work geared to the philosophy of simplifying reentry radiative heat transfer calculations without undue sacrifice in accuracy. In particular, reference (6) documents a numerical analysis for the viscous, nongray, radiating stagnation region shock layer, using approximate step models for the nongray, continuum, absorption coefficient of high temperature air. In fact, the explicit purpose of this analysis was to serve as a numerical instrument with which to investigate the engineering feasibility of such step model absorption coefficients. Early results[6,7] were obtained from the above analysis, using a now obsolete step model absorption coefficient. More recent numerical results[8] have been obtained using a much improved two-step model absorption coefficient, rationally constructed from existing quantum mechanical data; these recent results have shown that such a simple step model can be used in lieu of detailed spectral variations in order to obtain reasonable engineering results for shock layer, nongray, continuum radiative heat transfer.[8]

The closed-form formula for reentry radiative heat transfer derived in the present paper represents a further continuation of this engineering philosophy. The formula is not a numerical correlation of existing data; rather, it is derived on a rational physical basis. However, a hint with regard to a crucial physical assumption is revealed by close examination of some previously unpublished numerical results obtained during the preparation of reference (8), and makes

a simple derivation possible. In fact, the present analysis is an illustration of one role computer experiments play in engineering analyses, as described in a recent survey by Sichel.[9]

ANALYSIS

Background

As background for the following derivation, figures 1-3 show the extent to which radiative cooling and nongray self-absorption influence the radiating stagnation region shock layer and stagnation point heat transfer. These results were obtained from the viscous, nongray, continuum, radiating stagnation region analysis which is mentioned above,[8] and which is simply an extension of an earlier gray gas analysis by Howe and Viegas[10] to include nongray self-absorption. These analyses are well documented,[6-8,10] and therefore will not be described here. It is sufficient to state that they contain: (1) coupling of the radiative energy transport with the gasdynamic flow field, (2) nongray self-absorption (in the case of references (6) - (8)), (3) a fully viscous shock layer from the body to the bow shock, (4) local thermodynamic and chemical equilibrium, and (5) a self-similar solution limited to the stagnation region of a hypersonic, thin, radiating shock layer. In addition, the results in figures 1-3 were obtained with a two-step model, nongray, continuum absorption coefficient of high temperature air described in detail in reference (8); these results have been shown to agree favorably with the detailed spectral calculations of Hoshizaki and Wilson.[1,8] A sketch of the pertinent step model absorption coefficient is given in figure 4, where κ_1 and κ_2 are the vacuum ultraviolet (v-u-v) and

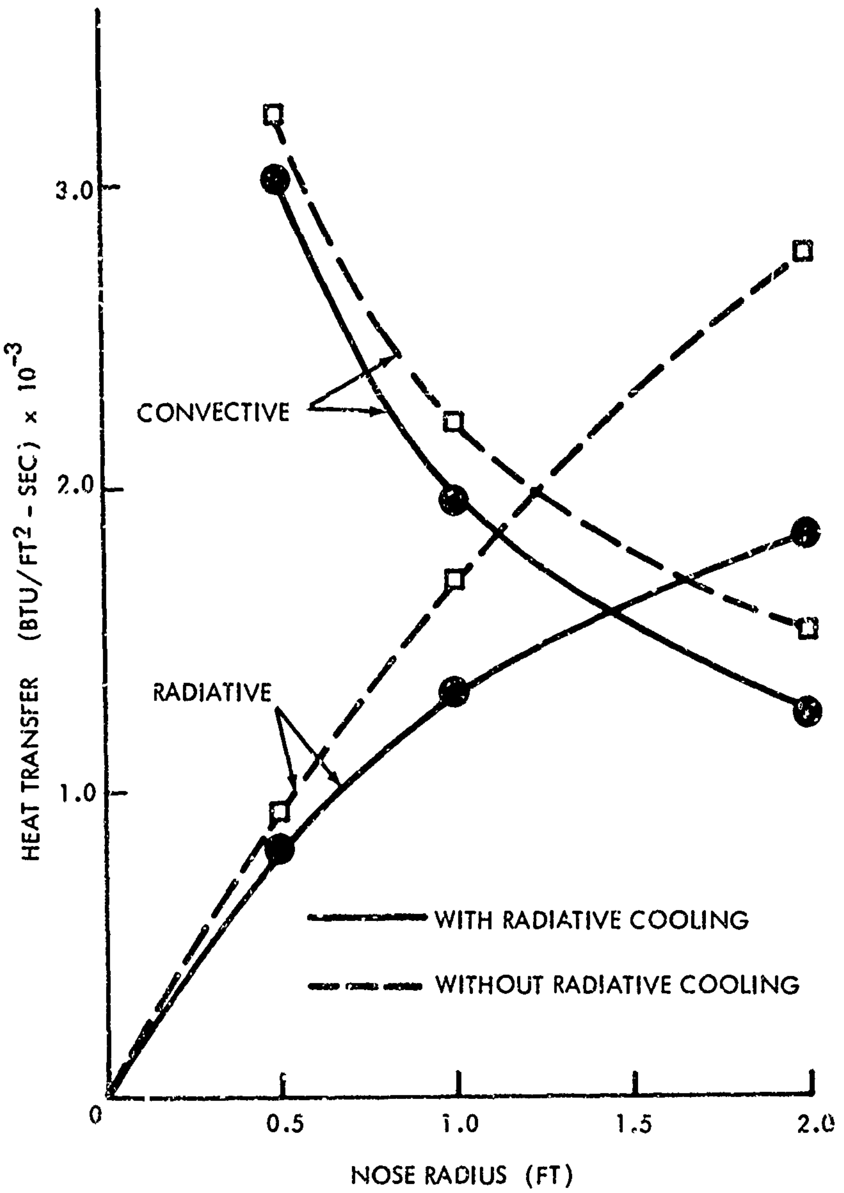

FIG. 1 EFFECT OF RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION ON STAGNATION POINT RADIATIVE AND CONVECTIVE HEAT TRANSFER. V_∞ = 50,000 FT/SEC; ALT = 200,000 FEET

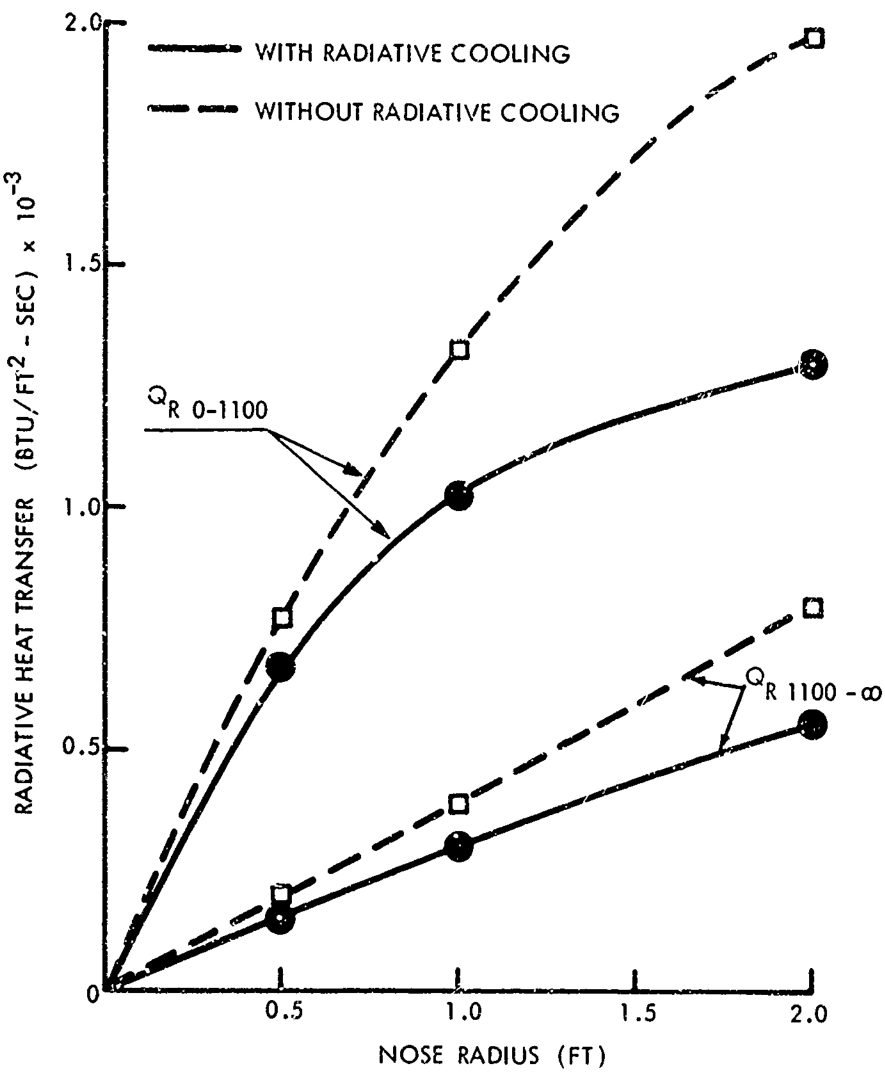

FIG. 2 EFFECT OF RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION ON THOSE PORTIONS OF STAGNATION POINT RADIATIVE HEAT TRANSFER ABOVE AND BELOW 1100 Å. V_∞ = 50,000 FT/SEC; ALT = 200,000 FEET

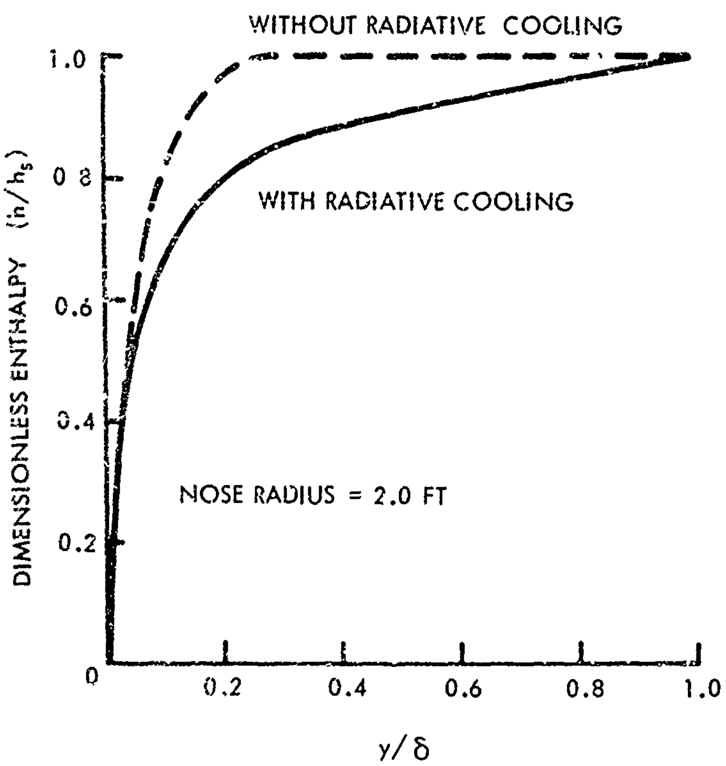

FIG. 3 EFFECT OF RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION ON THE STAGNATION REGION SHOCK LAYER ENTHALPY PROFILE. V_∞ = 50,000 FT/SEC; ALT = 200,000 FEET

long wave length absorption coefficients respectively,

$$\kappa_1 = \frac{\int_0^{1100\text{Å}} \kappa_\lambda B_\lambda d\lambda}{\int_0^{1100\text{Å}} B_\lambda d\lambda} = \frac{\int_0^{1100\text{Å}} \kappa_\lambda B_\lambda d\lambda}{B_1} \tag{1}$$

and

$$\kappa_2 = \frac{\int_{1100\text{Å}}^{\infty} \kappa_\lambda B_\lambda d\lambda}{\int_{1100\text{Å}}^{\infty} B_\lambda d\lambda} = \frac{\int_{1100\text{Å}}^{\infty} \kappa_\lambda B_\lambda d\lambda}{B_2} \tag{2}$$

κ_1 is obtained from the results of Hahne,[11] and κ_2 is obtained from one-half the radiance values of Nar[illegible]t al.[12] Reasonable correlations of κ_1 and κ_2 have been obtained as functions of the local gas density and temperature.[8]

Figures 1-3 compare results obtained with and without radiative cooling of the shock layer, i.e., with and without coupling of the radiative transport with the gasdynamic flow field. Nongray self-absorption is included in both cases. These results illustrate the strong influence of radiative cooling and nongray self-absorption on radiative heat transfer calculations; consequently, a closed-form engineering formula for superorbital, reentry, radiative heat transfer, Q_R, must take these effects into account. In the following analysis, an approximate, closed-form equation for Q_R is derived which indeed includes the combined effects of radiative cooling and nongray self-absorption. For the case of continuum radiation, Q_R will be considered as the sum of two contributions: (1) the v-u-v radiative heat transfer below 1100Å, $Q_{R_{0-1100}}$, and (2) the long wave length radiative heat transfer above 1100Å, $Q_{R_{1100-\infty}}$.

$$Q_R = Q_{R_{0-1100}} + Q_{R_{1100-\infty}} \qquad (3)$$

This division of Q_R into two distinct (but coupled) parts is suggested by the spectral variation of the continuum radiation properties for high temperature air,[1,2] and is consistent with the use of an approximate step model absorption coefficient.[8] We will now proceed with the derivation of closed-form expressions for $Q_{R_{0-1100}}$ and $Q_{R_{1100-\infty}}$ which allow rapid but reasonably accurate calculations of these contributions. Extension to the case of atomic line radiation will be discussed in a subsequent section.

<u>Vacuum Ultraviolet Contribution</u>

$Q_{R_{0-1100}}$ is markedly affected by radiative cooling and strong self-absorption within the shock-heated gas.[1-8] However, these two effects combine in a manner that allows a crucial physical assumption to be made, subsequently leading to a simple expression for $Q_{R_{0-1100}}$. To illustrate this point, figure 5 shows the effect of radiative cooling and nongray self-absorption on the local values of the product $B_1\epsilon_2$ as a function of local v-u-v optical length, τ_1, through the shock layer, where $B_1 = \int_0^{1100\text{Å}} B_\lambda d\lambda$, ϵ_2 is the integro-exponential function of second order, and $\tau_1 = \int_0^y \kappa_1(y)dy$. The areas under the curves in figure 5 are proportional to the v-u-v radiative heat transfer to the surface if we assume one-dimensional radiative transfer through the shock layer, a cold, non-emitting, black surface, and if we imply that the gas absorption coefficient is adequately represented by figure 4 (as validated in reference (8)). That is [6,13]

$$Q_{R_{0-1100}} = 2\pi\int_0^{\tau_{1s}} B_1(\tau)\epsilon_2(\tau)\,d\tau \qquad (4)$$

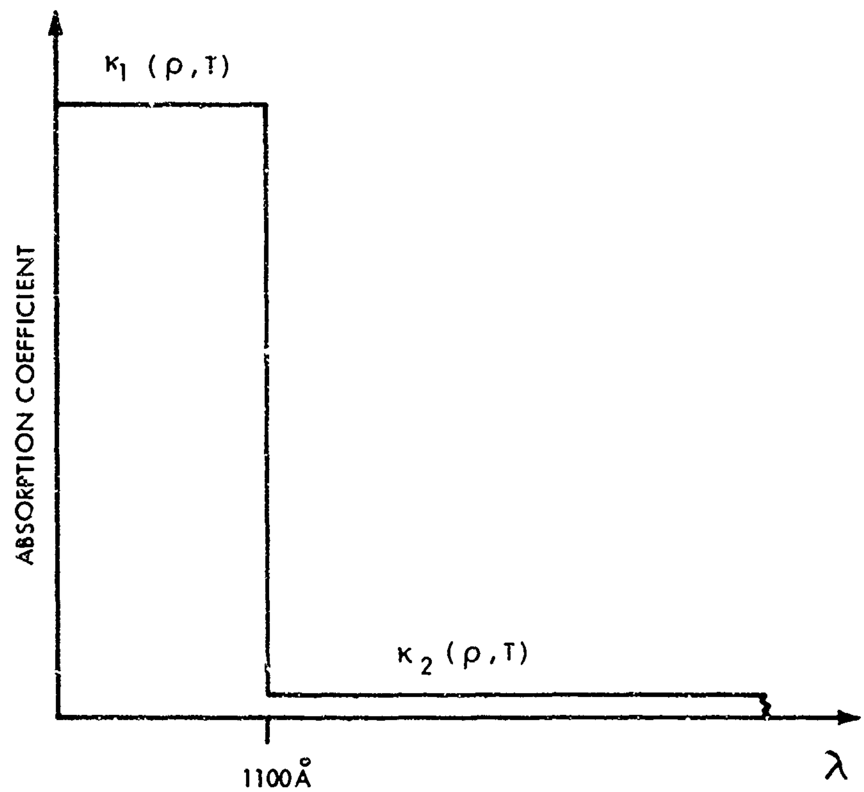

FIG. 4 SCHEMATIC OF NONGRAY CONTINUUM STEP MODEL ABSORPTION COEFFICIENT

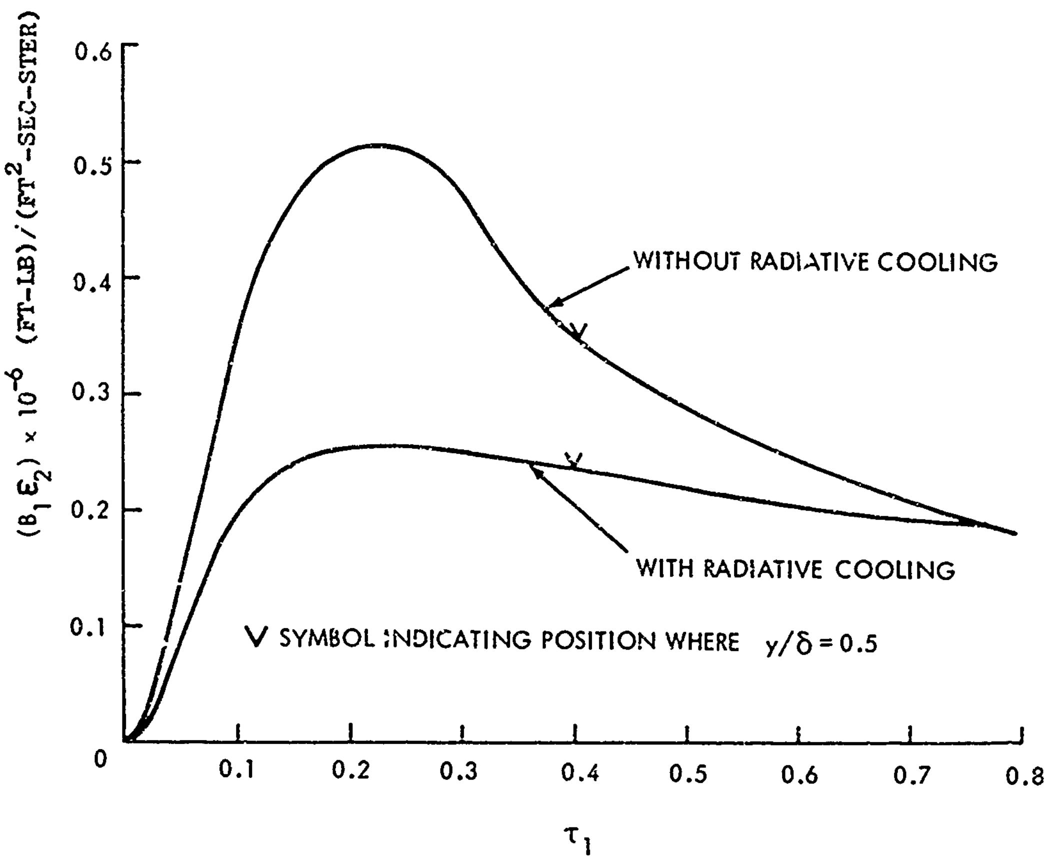

FIG. 5 EFFECT OF RADIATIVE COOLING AND NONGRAY SELF-ABSORPTION ON THE DISTRIBUTION OF $B_1 \epsilon_2$ THROUGH THE STAGNATION REGION SHOCK LAYER AS A FUNCTION OF τ_1. V_∞ = 50,000 FT/SEC; ALT = 200,000 FEET; R=2.0 FEET

The previously unpublished numerical results shown in figure 5, which were obtained during the preparation of reference (8), indicate that the local maximum of $B_1\epsilon_2$ is predominantly governed by radiative cooling, whereas the decrease in $B_1\epsilon_2$ for larger values of τ_1 is due to self-absorption (i.e., small values of ϵ_2). The influence of the thermal boundary layer near the wall is responsible for the decrease in $B_1\epsilon_2$ at smaller values of τ_1. It is important to notice in figure 5 that radiative cooling and self-absorption flatten the distribution of $B_1\epsilon_2$ through the shock layer. This trend is further supported by the results of figure 6, which graphically shows the increasing effect of radiative cooling and self-absorption as R is increased. This flattening of the $B_1\epsilon_2$ curve hints very strongly at a simple approximation for $Q_{R_{0-1100}}$; namely, the area under the curve appears to be reasonably approximated by a rectangle of height $(B_1\epsilon_2)_s$ and of length τ_{1s}, as shown by the dotted lines in figure 6. (The subscript s implies conditions evaluated immediately behind the bow shock.) With this approximation, equation (4) becomes

$$Q_{R_{0-1100}} = 2\pi (B_1\epsilon_2)_s \, \tau_{1s} \tag{5}$$

This expression can be readily evaluated knowing κ_{1s} and the shock detachment distance, δ, as will be outlined in a subsequent section. In addition, the results shown in figure 6 indicate that the above assumption, and therefore equation (5), becomes more realistic for increasing values of R, where the effects of radiative cooling and nongray self-absorption become stronger, and where the thermal boundary layer becomes a smaller fraction of the total shock layer.

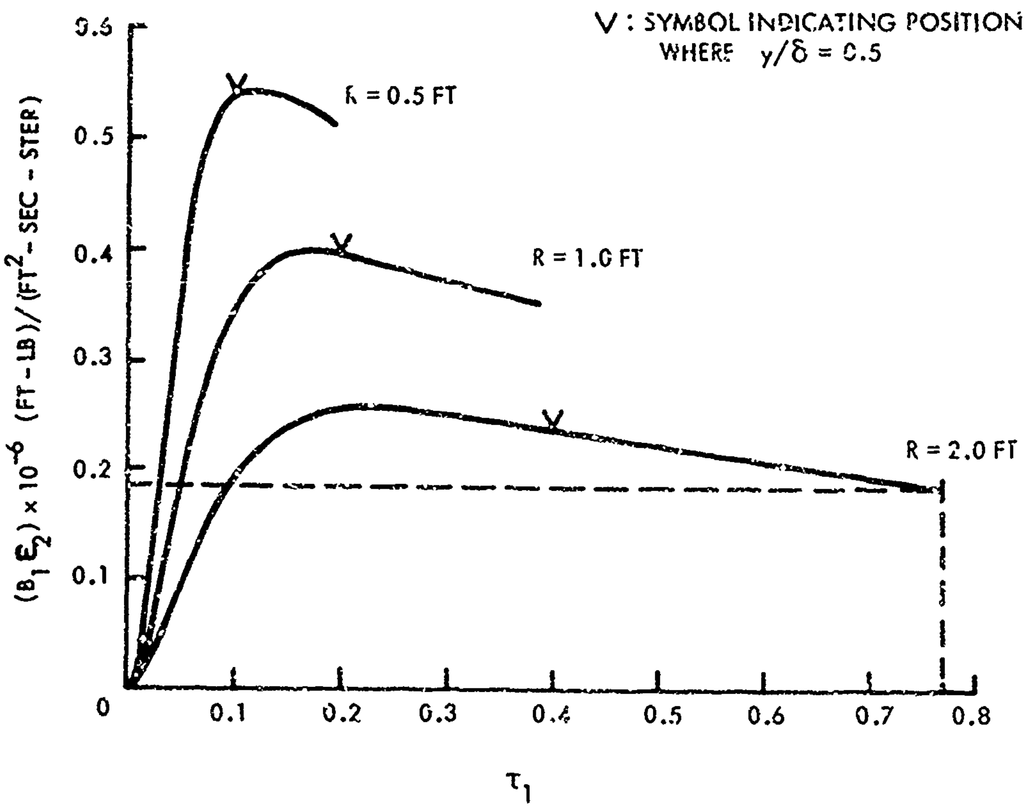

FIG. 6 VARIATION OF $B_1 \epsilon_2$ WITH τ_1 THROUGH THE STAGNATION REGION FOR VARIOUS NOSE RADII. V_∞ =50,000 FT/SEC; ALT=200,000 FEET

Long Wave Length Contribution

In contrast to the strong self-absorption occurring in the v-u-v wave length range, the shock layer is optically thin for long wave length continuum radiation,[1-8] thus providing an advantage in obtaining a simple expression for $Q_{R_{1100-\infty}}$. However, $Q_{R_{1100-\infty}}$ is affected by radiative cooling of the shock layer due to radiative emission from all wave length ranges, and this effect must be taken into account.

For a transparent, constant property, equilibrium shock layer,[14] the stagnation point radiative heat transfer is given by $E_s\delta_{AD}/2$, where E_s is the radiative energy emitted by the shock-heated gas per unit time and volume evaluated for equilibrium conditions behind the normal bow shock, and δ_{AD} is the shock detachment distance. However, if the transparent shock layer is cooled due to radiative emission, the stagnation point radiative heat transfer is reduced,[15] and is given by $(E_s\delta_{AD}/2)f(\Gamma)$, where $f(\Gamma) \leq 1$ and is a function of the radiation loss parameter.

$$\Gamma = \frac{E_s\delta_{AD}}{\rho_\infty V_\infty^3/2} \tag{6}$$

A curve for $f(\Gamma)$ has been obtained by Hoshizaki (figure 9 of reference (15)); as indicated in reference (6), this curve is apparently independent of the gas radiative properties.

From the above experience, it appears reasonable that $Q_{R_{1100-\infty}}$ can be expressed as

$$Q_{R_{1100-\infty}} = (E_{2s}\delta_{AD}/2)f(\Gamma_{eff}) \tag{7}$$

where $E_{2s} = 4\pi(\kappa_2 B_2)_s$ and $B_2 = \int_{1100}^{\infty} B_\lambda d\lambda$. Thus,

$$Q_{R1100-\infty} = 2\pi(\kappa_2 B_2)_s \delta_{AD}\ f(\Gamma_{eff}) \tag{8}$$

Γ_{eff} is defined here as an "effective" radiation loss parameter which represents the ratio of the radiative energy flux out of a constant property, uncoupled, nongray shock layer to the flux of enthalpy convected into the shock layer from the free stream through the bow shock. Γ_{eff} must contain the influences of both the strongly absorbed v-u-v as well as the relatively transparent long wave length ranges. Because the physical effect of self-absorption is to trap some of the energy within the absorbing shock layer which would otherwise be lost if the gas were transparent, $\Gamma_{eff} < \Gamma$. Also, because of the different nature of the two wave length regions, it is reasonable to represent Γ_{eff} as the sum of two terms

$$\Gamma_{eff} = \Gamma_1 + \Gamma_2 \tag{9}$$

where Γ_1 and Γ_2 apply to the v-u-v and long wave length regions respectively. An expression for Γ_2 is immediately suggested by equation (6) applied for the (transparent) long wave length region:

$$\Gamma_2 = E_{2s}\delta_{AD}/(\rho_\infty V_\infty^3/2) = 4\pi(\kappa_2 B_2)_s \delta_{AD}/(\rho_\infty V_\infty^3/2) \tag{10}$$

An expression for Γ_1 is suggested by the results of reference (4), which indicate that an "effective" value of the radiation loss parameter including self-absorption for a gray gas can be approximated by the product $\left[\exp(-b\tau_s)\right]\Gamma$, where τ_s is the gray optical thickness of the shock layer, and b is a constant. Applying this

result to the v-u-v region for the present analysis, and approximating b by unity as suggested by inspection of the numerical results of reference (8), the following expression for Γ_1 is obtained:

$$\Gamma_1 = \frac{e^{-\tau_{1s}} E_{1s}\delta_{AD}}{\rho_\infty V_\infty^3/2} = \frac{4\pi e^{-\tau_{1s}}(\kappa_1 B_1)_s \delta_{AD}}{\rho_\infty V_\infty^3/2} \tag{11}$$

Note that Γ_1 approaches the correct optically thin expression as τ_{1s} becomes small. However, it is conceded that a more appropriate expression for Γ_1 might be obtained with further study. In fact, equation (11) shows that Γ_1 approaches zero as δ_{AD} (and thus τ_{1s}) becomes very large, whereas it is physically reasonable that Γ_1 should approach a finite limit. Consequently, as τ_{1s} becomes large, the v-u-v radiative energy loss through the bow shock is not entirely accounted for in equation (11). (On the other hand, it will be shown later that the v-u-v radiative energy loss in the opposite direction, towards the surface, does indeed approach zero as τ_{1s} becomes very large.) Nevertheless, for practical values of shock layer thickness, equation (11) is considered sufficient for the present purpose. Therefore, from equations (9), (10) and (11),

$$\Gamma_{eff} = \frac{4\pi\left[e^{-\tau_{1s}}(\kappa_1 B_1)_s + (\kappa_2 B_2)_s\right]\delta_{AD}}{\rho_\infty V_\infty^3/2} \tag{12}$$

Consequently, equation (8), with Γ_{eff} evaluated as equation (12), allows a simple evaluation of $Q_{R_{1100-\infty}}$ which takes into account radiative cooling from all wave lengths.

Application of Formula

The complete expression for the total stagnation point, continuum, nongray, radiative heat transfer is obtained from equations (3), (5) and (8) as:

$$Q_R = 2\pi(B_1\mathcal{E}_2)_s\,\tau_{1s} + 2\pi\left[(\kappa_2 B_2)_s\,\delta_{AD}\right]f(\Gamma_{eff}) \qquad (13)$$

with Γ_{eff} given by equation (12). For a given reentry trajectory point (ρ_∞ and V_∞) and a given nose radius, R, equation (13) can be rapidly evaluated by hand in a straightforward fashion as outlined below.

(1) Obtain from normal shock wave tables (such as reference (16)) the equilibrium density and temperature, ρ_s and T_s, behind the bow shock. For ρ_s and T_s, obtain κ_{1s} from Hahne's[11] tables and κ_{2s} from one half the radiance values of Nardone et al.[12] Alternatively, use the approximate correlations for κ_{1s} and κ_{2s} available from reference (8).

(2) Obtain δ_{AD} from hypersonic flow theory. The following expression is recommended,[17]

$$\delta_{AD}/R = \frac{\varepsilon}{1 + (2\varepsilon)^{\frac{1}{2}}} \qquad (14)$$

where $\varepsilon = \rho_\infty/\rho_s$.

(3) Calculate τ_{1s}; $\tau_{1s} = \kappa_{1s}\delta_{AD}$. Then find $\mathcal{E}_2(\tau_{1s})$ from existing tables of the integro-exponential function.[13,18]

(4) For T_s, obtain B_1 and B_2 from figure 7, or from numerical evaluation of their definitions.

(5) Evaluate Γ_{eff} from equation (12).

(6) Obtain $f(\Gamma_{eff})$ from figure 8, or from the more extended curve in figure 9 of reference (15).

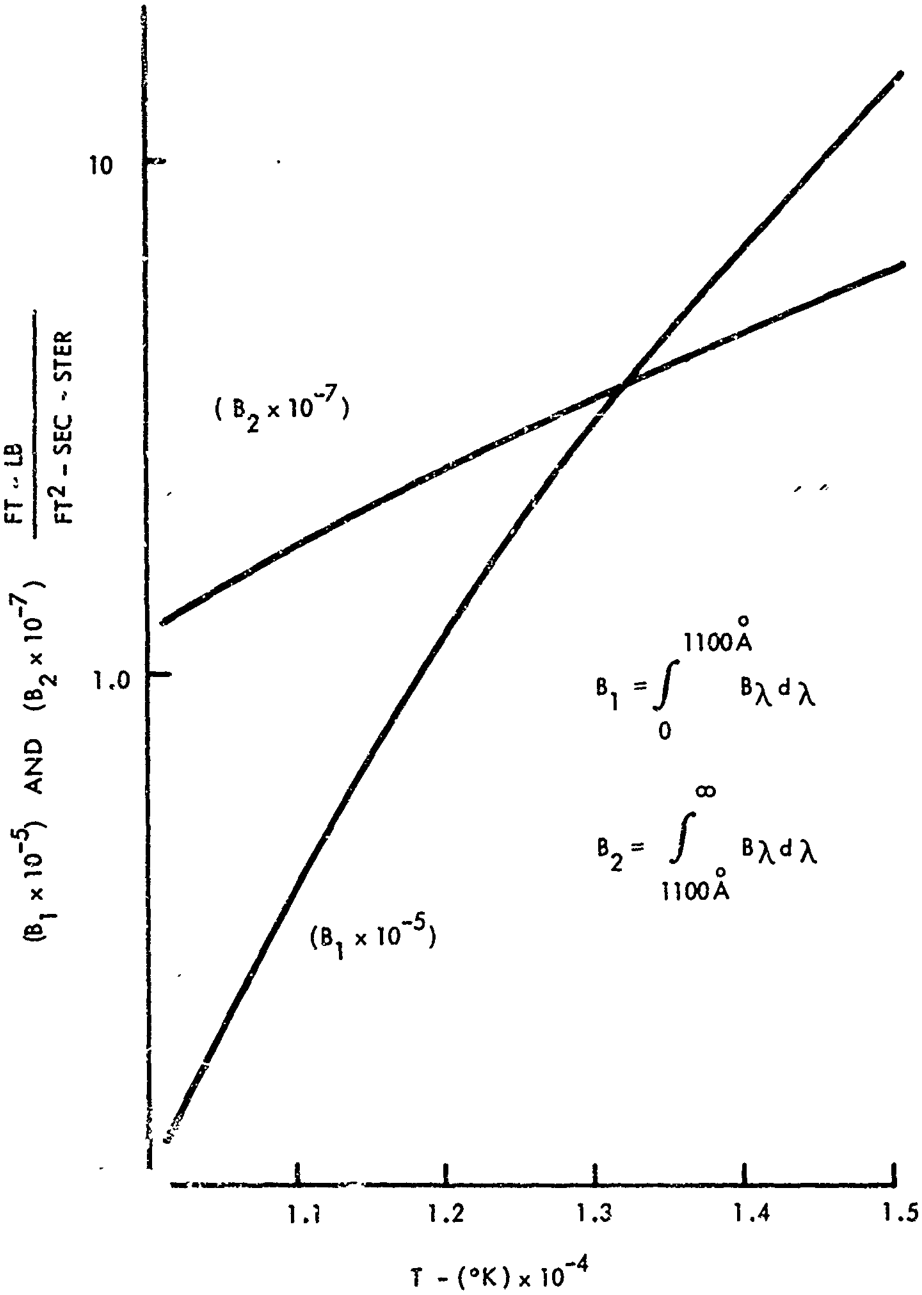

FIG. 7 B_1 AND B_2 AS FUNCTIONS OF TEMPERATURE

(7) Evalute the stagnation point continuum, nongray, radiative heat transfer from equation (13).

The above development has been carried out for the case of continuum radiation with the continuum absorption coefficient for high temperature air reasonably approximated by a step model such as that shown in figure 4. However, this analysis can be extended to include atomic line radiation as reasoned below. Recent measurements[19-21] of atomic line radiation from high temperature air have indicated that the line intensities are strongly self-absorbed. In addition, Nerem[5] is currently working on a four-step model absorption coefficient which includes atomic line as well as continuum contributions. Also, Olstad[22] at NASA Langley is developing a combined atomic line and continuum step model absorption coefficient. Consequently, it appears that equation (13) can be extended in a straightforward manner to include atomic line radiation by considering a multi-step absorption coefficient model and by evaluating the contribution to Q_R from each step in the manner prescribed above. Of course, Γ_{eff} should also be extended to account for each step. These extensions are not explicitly made in the present paper due to the current unavailability of a precise step model absorption coefficient which includes atomic line radiation.

RESULTS

Equation (13) compares favorably with existing numerical data for stagnation point, nongray, continuum, radiative heat transfer, as can be seen from figure 9 and Table 1. Figure 9 compares the values of Q_R obtained from equation (13) with the numerical results

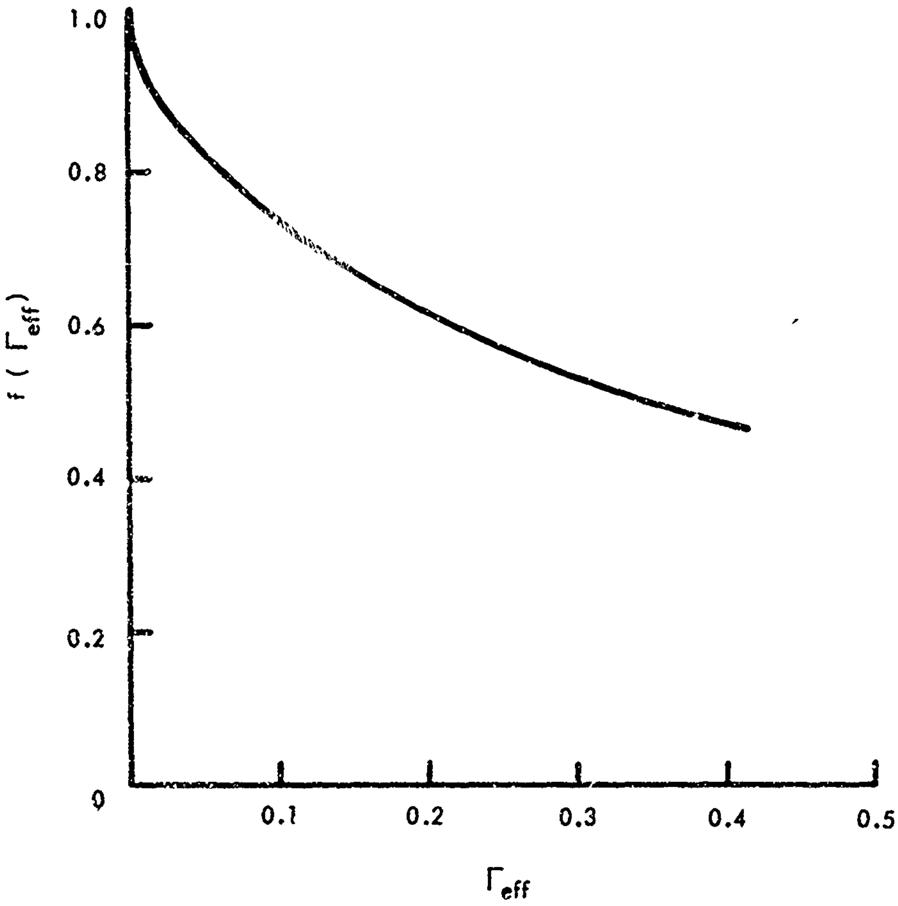

FIG. 8 $f(\Gamma_{eff})$ AS A FUNCTION OF Γ_{eff}

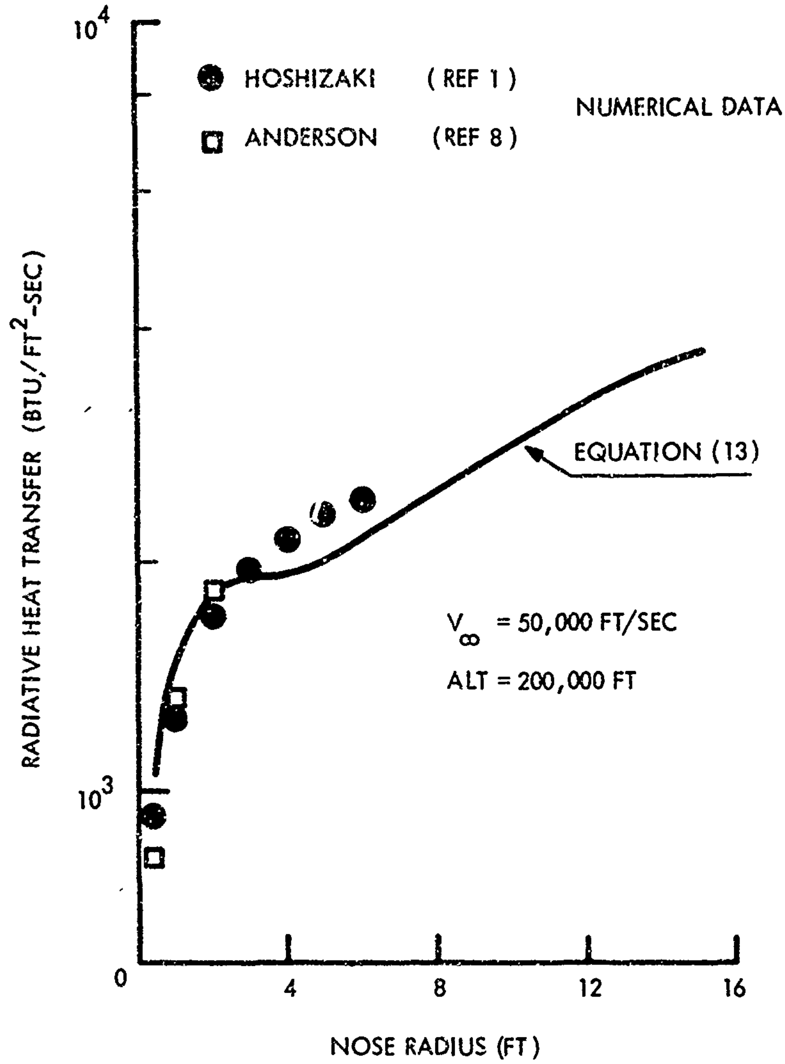

FIG. 9 COMPARISON OF ENGINEERING FORMULA WITH NUMERICAL RESULTS FOR STAGNATION POINT CONTINUUM RADIATIVE HEAT TRANSFER

		RESULTS FROM EQUATION (13)				COMPARISON WITH EXISTING NUMERICAL RESULTS	
TRAJECTORY POINT	R FT	Γ_{eff}	$Q_{R_{O-1100}}$ BTU/FT^2-SEC	$Q_{R_{1100-\infty}}$ BTU/FT^2-SEC	Q_R BTU/FT^2-SEC	Q_R (ANDERSON REF. 8) BTU/FT^2-SEC	Q_R (HOSHIZAKI AND WILSON, REF. 1) BTU/FT^2-SEC
V_∞=50,000 FT/SEC ALTITUDE=200,000 FT	0.5	0.0567	882	185	1067	820 (30%)*	930 (15%)
	1	0.097	1138	337	1475	1330 (11%)	1250 (18%)
	2	0.1414	1220	620	1840	1855 (0.8%)	1700 (8%)
	3	0.161	1010	886	1896	--	1950 (3%)
	4	0.1685	749	1181	1930	--	2180 (11%)
	5	0.172	552	1468	2020	--	2300 (12%)
	6	0.175	392	1748	2140	--	2400 (11%)
	9	0.1958	138	2550	2688	--	--
	12	0.234	35.5	3180	3215	--	--
	15	0.282	33.7	3730	3764	--	--
V_∞=41,000 FT/SEC ALTITUDE=180,000 FT	1	0.0444	417	275	692	810 (14%)	715 (3%)
	2	0.0557	286	535	821	1121 (27%)	--
V_∞=37,000 FT/SEC ALTITUDE=230,000 FT	4	0.0277	44.5	11.3	55.8	--	40 (39%)

* NUMBERS IN PARENTHESIS INDICATE PERCENTAGE DIFFERENCE BETWEEN EQ (13) AND NUMERICAL RESULTS

TABLE I TABULATED INFORMATION FROM ENGINEERING FORMULA AND COMPARISON WITH NUMERICAL RESULTS

of Hoshizaki and Wilson,[1] and Anderson,[8] for the trajectory point: $V_\infty = 50{,}000$ ft/sec; altitude = 200,000 feet. Table 1 presents a more detailed comparison between these data, including additional trajectory points. Even though the existing published numerical data does not extend beyond a nose radius of 6 feet, figure 9 shows results from equation (13) up to R = 15 feet, which is characteristic of Apollo size reentry vehicles. These comparisons show that equation (13) is apparently a reasonable engineering formula for stagnation point radiative heat transfer taking into account the effects of radiative cooling and nongray self-absorption within the shock layer. In addition, the shape of the curve shown in figure 9 reflects the physical fact that Q_R is dominated by the v-u-v contribution at small R, whereas for large R the v-u-v radiation is strongly self-absorbed and the long wave length radiative heat transfer is by far the dominant contribution.[1,4-8] These trends can be seen in detail in Table 1. In fact, for large nose radii (R on the order of 15 feet) the present approximate results indicate that $Q_{R_{0-1100}}$ may be on the order of one percent of Q_R. These proportions remain to be verified by detailed numerical and/or experimental results for large nose radii; however, the results of Nerem and Carlson[5] for shock tube end-wall radiative heat transfer behind a strong reflected shock wave in air indicate that such proportions between $Q_{R_{0-1100}}$ and $Q_{R_{1100-\infty}}$ are reasonable for thick shock layers. Of course, on a physical basis, the striking reduction of $Q_{R_{0-1100}}$ for large nose radii is due to the strong v-u-v self-absorption, thus attenuating $Q_{R_{0-1100}}$, and to radiative cooling of the shock layer, which reduces the local radiative emission as well as shifts the peak of the Planck black body curve to longer wave lengths.

An interesting comparison is also obtained by applying equation (13) to the end-wall radiative heat transfer behind a strong reflected shock wave in a shock tube; for this case, $\delta_{AD} = W_R t$, where W_R is the reflected shock wave velocity and t is the time after reflection. Figure 10 compares the approximate formula with the shock tube results of Nerem and Golobic[5] for the case of a reflected shock wave produced by an initial incident shock velocity of 8.85 mm/μ sec and initial driven tube pressure of 1 mm Hg. The comparison is again favorable, even though in this case somewhat dissimilar quantities are being compared. That is, equation (13) with κ_{1s} and κ_{2s} evaluated from references (11) and (12) respectively, as described above, represents continuum, nongray, radiative heat transfer over the entire wave length spectrum. On the other hand, the shock tube results of Nerem and Golobic give measured radiative heat transfer for $0.17 < \lambda < 6\mu$, and contain contributions from atomic line as well as continuum radiation. These dissimilarities are to some degree mutually compensating, however, the relatively favorable agreement shown in figure 10 should be construed as somewhat fortuitous. Nevertheless, the point is made that equation (13) can be used to predict end-wall as well as stagnation point radiative heat transfer.

CONCLUSIONS

An approximate, closed-form equation has been developed which allows rapid calculation of reentry, stagnation point, radiative heat transfer taking into account the effects of radiative cooling and nongray self-absorption within the shock layer. This equation

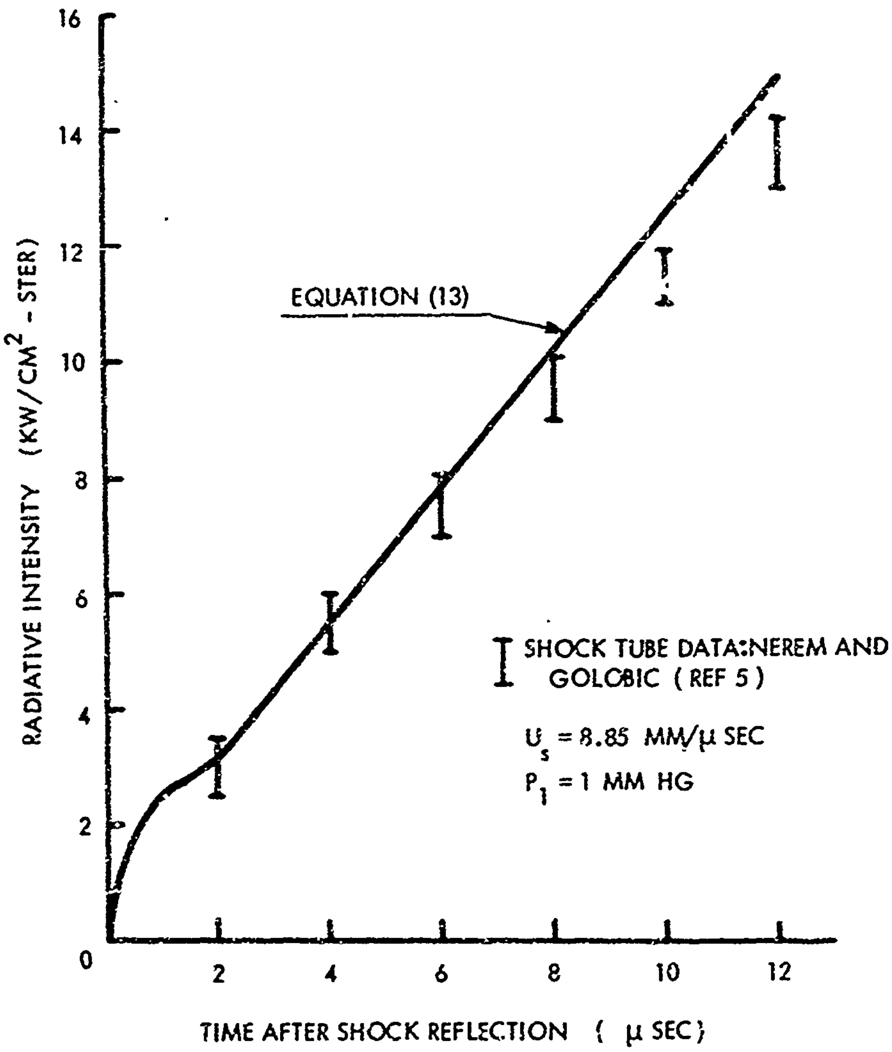

FIG. 10 COMPARISON OF ENGINEERING FORMULA WITH MEASURED SHOCK TUBE END-WALL RADIATIVE HEAT TRANSFER RATES BEHIND A REFLECTED SHOCK WAVE

is not a correlation of existing numerical data; rather, it has been derived on a rational physical basis. However, a hint with regard to a crucial physical assumption was revealed by close examination of numerical results; namely, that the product $B_1 \epsilon_2$ is reasonably constant through the shock layer due to the combined influences of radiative cooling and nongray self-absorption. The resulting formula compares favorably with existing numerical calculations of stagnation point, nongray, continuum radiative heat transfer. In addition, a suggestion is made for an extension of the formula to include atomic line as well as continuum radiation, predicated upon the future development of a step model absorption coefficient which represents both types of radiation. Also, in addition to the stagnation point case, the formula is shown to predict end-wall radiative heat transfer behind a strong reflected shock wave. Finally, the formula provides a rapid means of obtaining, by hand, reasonably accurate engineering estimates of reentry radiative heat transfer including radiative cooling and nongray self-absorption, thus circumventing lengthy computer solutions. Of course, the formula is not intended to replace numerical calculations when very high accuracy is desired.

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A closed-form equation is derived for stagnation point reentry radiative heat transfer accounting for the combined effects of radiative cooling and nongray self-absorption within the shock layer. The equation can be applied for both continuum and atomic line radiation. In addition, the equation is shown to agree favorably with existing numerical data for stagnation point, continuum radiative heat transfer for a wide variety of conditions. Also, the equation is shown to apply to the end-wall radiative heat transfer behind a strong reflected shock wave in a shock tube. Finally, the equation provides a rapid means of obtaining, by hand, reasonably accurate engineering estimates for reentry radiative heat transfer including shock layer radiative cooling and nongray self-absorption.

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